Rational Embeddings of Convex Polyhedra

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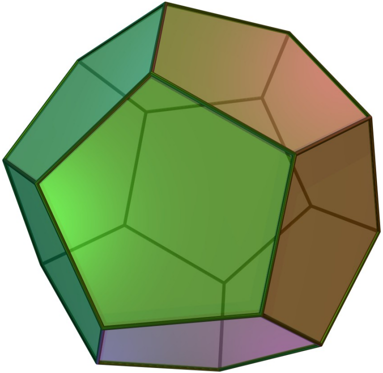
Question

Do all convex polyhedra have embeddings into $\mathbb{R}^3$ with all rational edge lengths?



Theorem (Steinitz)

A graph is the edge graph of a polyhedron iff it is a planar and 3-connected graph.

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Theorem (Sun)

All simplicial polyhedra have embeddings with all side lengths rational.

Previous Results and Conjectures (continued)

Two dimensions:

Conjecture (Harborth)

All planar graphs have embeddings with all edge lengths rational

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Does there exist a dense set of points with pairwise rational distances in $\mathbb{R}^3$?

Question

Does there exist a dense subset of the unit sphere with pairwise rational distance?

- Polyhedra with all vertices on a sphere
- Non-inscribable polyhedra exist!

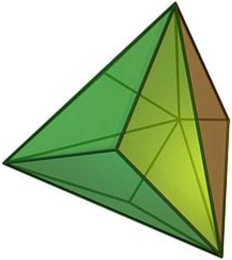

Figure: A Triakis tetrahedron, with no embedding on a sphere

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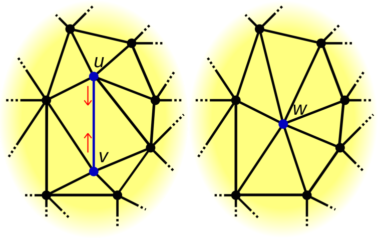
- Probabilistic, inductive approach
- Edge contraction shrinks a simplicial polyhedron to a smaller one



Conjecture

A randomly chosen simplicial polyhedron on n vertices is inscribable with probability at least $(\frac{2}{9})^n$

Future work

- Stronger bounds
- Not necessarily simplicial polyhedra
- Other problems concerning the embeddings of polyhedra

Thanks to PRIMES for providing this opportunity, Ravi Vakil for suggesting this project, and my mentor Sheela Devadas